Changing Regulatory Capital to Include Liquidity and Management Intervention

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This article presents a practical approach for incorporating the effects of illiquidity and management response lags in settling regulatory capital levels for market risk. The Bank for International Settlements (Basle Committee on Banking Supervision [1996]) sets the capital level that banks must hold against market risks according to the formula:[1]

$$\text{Capital} = 3\sqrt{10}\,\text{VaR}_{99\%}$$

where $\text{VAR}_{99\%}$ is the ninety-ninth percentile one-day value at risk (VaR). Although this formula represents a significant improvement over previous methods of setting capital, significant limitations remain. We present a straightforward approach for amending regulatory capital to mitigate two of these limitations: liquidity constraints and response lags in management intervention.

Capital should be a function of the effectiveness of limit management and market liquidity because actively managing limits and positions can significantly reduce the risk of a trading operation. Illiquidity of positions in a given portfolio, however, reduces the effectiveness of management's best attempts to modify exposures and therefore increases portfolio risk. Our approach quantifies this increase in portfolio risk and the associated increase in capital required to maintain the bank's creditworthiness.

Our recommendations are summarized as follows: the capital level required should be modified to include a liquidity scaling factor, typically between 0.9 and 1.8. This factor, α, depends on the liquidity of the position and the effectiveness of the institution in managing its limits. It serves to scale up (or down) the capital level required. Specifically, the regulations should be amended as follows:

$$\text{Capital} = \alpha 3\sqrt{10}\,\text{VaR}_{99\%}$$

For the approach used in this article, the derived values for α are given in Exhibit 1. The operating principle demonstrated in Exhibit 1 can be articulated as follows: *Strictly-managed liquid positions should require less capital than loosely-managed illiquid positons.*

The "Closeout Days" (T) represent the time required to liquidate the position. The "Reporting Period" (R) is defined as the time period between limit resets. Since the relative liquidity scaling factor a is significantly less than the "square root of T" adjustment that is currently in common use to report the effect of liquidity on VaR, adopting the scaling factor a could yield added cost efficiencies in managing the capital of a trading operation.[2]

First, we describe the approach taken to obtain the values for α shown in Exhibit 1. The rest of the article proceeds as follows: Second, we describe our broad approach. Third, we focus specifically on illiquidity.

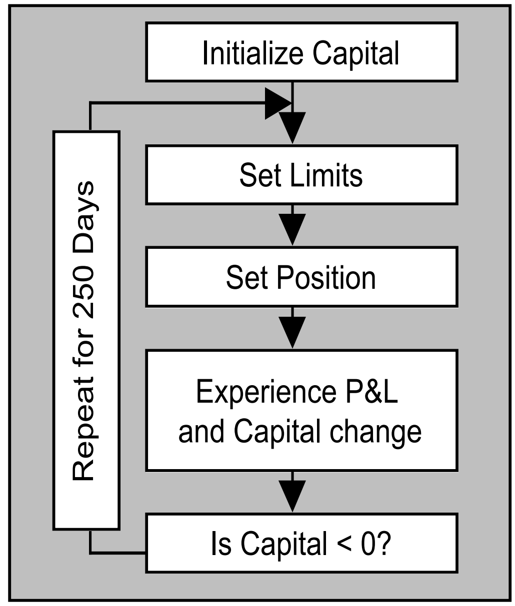
EXHIBIT 1

Relative Capital Required (a) for Different Combinations of Liquidity and Limit Mangement

Reporting Period (R) \ Closeout Days (T)	1	5	10	21	62	∞
1	0.92	0.96	0.99	1.02	1.11	**1.79**
5	0.95	0.98	1	1.04	1.12	**1.79**
10	0.98	1.00	1.03	1.06	1.17	**1.79**
21	1.04	1.05	1.07	1.09	**1.21**	**1.79**
62	**1.21**	**1.21**	**1.21**	**1.21**	**1.38**	**1.79**
250	**1.79**	**1.79**	**1.79**	**1.79**	**1.79**	**1.79**

EXHIBIT 2

Interaction Between Limits, Positions, and Market Movements



Fourth, we examine the impact of speed of management intervention. Fifth, applications to capital management are considered. Finally, we offer a conclusion and point to directions for further research.

APPROACH

We deliberately stylize the approach described here to ensure that we do not obscure the essential message: regulatory capital can, and should, be a function of liquidity and effectiveness of limit management practices.

Our approach uses Monte Carlo evaluation to randomly simulate a year of trading activity, and thereby determines the annual probability of default. One may think of this as a simulation laboratory in which to experiment with different management intervention policies, limit-setting rules, and varied liquidity environments. The institution is considered to have defaulted if, during the year, the value of its assets falls below the value of its liabilities, i.e., the available economic capital is less than zero. The trading activities include the following elements: 1) setting the initial capital level, 2) setting limits, 3) changing position sizes, and 4) experiencing daily market returns. This sequence is illustrated in Exhibit 2.

In this article, we make the following assumptions (for technical details, please see the appendix):

- The limit structure is based on VaR.[3] The VaR limit is defined by $\text{Capital} = m\,3\sqrt{10}\,\text{VaRLimit}$. Here "Capital" is defined as the capital available (value of assets minus value of debt) and m is an exploratory parameter that we will vary to find what happens with more or less capital. If m is small, the amount of capital held will be small relative to the allowed VaR. Alternatively, one could say that if m is small, the amount of risk allowed is large relative to the capital available. α is the specific value of m that assures that the probability of default does not exceed the "critical" (i.e. maximum acceptable) value.
- The daily market returns are sampled using a jump-diffusion process from a probability distribution with the same mean, standard deviation, and degree of kurtosis[4] as the S&P 500 index. This produces a distribution with the tail probabilities weighted to account for a higher occurrence of extreme events than occur in a normal distribution.
- The position is leveraged, therefore the daily P&L is reduced by the cost of debt.[5]

An example of running the simulation is shown in Exhibits 3 and 4. Exhibit 3 shows the paths of five different simulations. Each simulation begins with $1 of capital. Over the course of the trading period, capital will increase (decrease) as a function of the trading profit (loss). The lowest path illustrates a scenario in which a series of losses reduces capital, until by the end of the period, only $0.30 capital remains. Note that on the lowest path, the absolute level of volatility is low because the limits are tight-

EXHIBIT 3
Example of Five Simulations

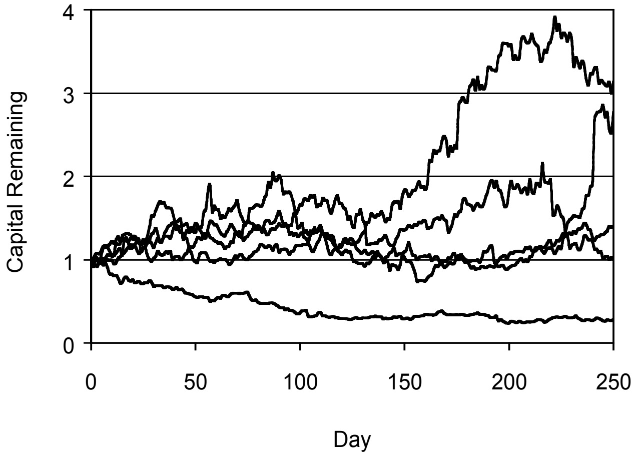
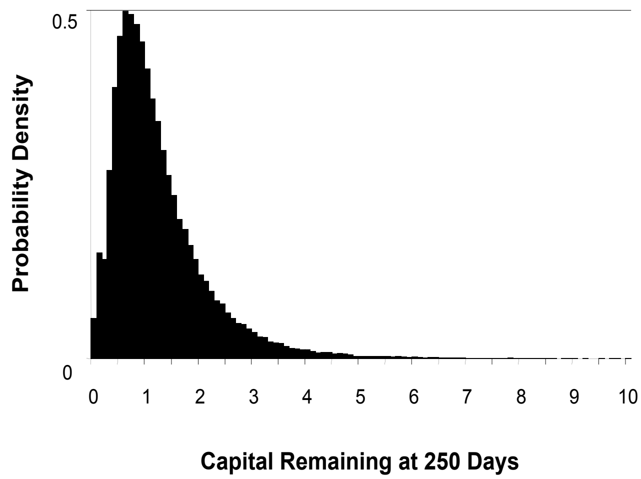


EXHIBIT 4
Distribution of Final Capital



ened as capital is lost. In contrast, the top path shows a trading period in which the value of the position is tripled to \$3. In each of these five simulations, the bank remains solvent, i.e., the level of capital remains positive.

By running 100,000 such simulations, we obtain Exhibit 4, which shows the probability distribution of the final capital level at 250 days, a trading year. In Exhibit 4, there is a 5% probability that the capital level will exceed or be equal to \$3 by the end of the period. The probability that capital declines to less than \$0.3 is also approximately 5%. The 0.57% probability that capital will equal zero at the end of the period suggests a 99.43% expected survival probability. The skew in the distribution is caused by the expansion (contraction) of limits as capital increases (decreases).

The base case example shown in Exhibits 3 and 4 makes the following three assumptions:

1. The ratio between the VaR limits and the available capital is set according to the current BIS regulation (m = 1), i.e.,

$$\text{Capital} = 1 \times 3\sqrt{10}\,\text{VaRLimit}$$

2. The market is perfectly liquid (T = 1).
3. The limits are reset each day (R = 1).

Exhibit 5 illustrates the effect of changing the multiplier, m. As one would expect, smaller values of m result in a higher probability of default. With m = 0.5 the capital is 50% of the capital required by the BIS, i.e.,

$$\text{Capital} = 0.5 \times 3\sqrt{10}\,\text{VaRLimit}$$

With m = 0.5, the probability of default is 382 basis points. With m = 1, capital corresponds with the Bank for International Settlements (BIS) requirement and the probability of default decreases to 57 bp. With m = 1.5, the probability of default falls to 3 bp. Although this result initially seems reassuring, it excludes the effects of asset or market illiquidity and slow management response.

ILLIQUIDITY

Illiquidity manifests itself in two ways: 1) a wide bid-ask spread and 2) significant movement in the mid-price when large blocks are sold in the market. Wide bid-ask spreads are generally observed in markets with limited information and few active participants.[6] This result suggests that the VaR should be increased by the volatility of the bid-ask spread. In this article, however, we are concerned with the other aspect of illiquidity: the effect of supply and demand on the mid-price when large blocks are traded. If the trader attempts to sell a position which is large relative to market demand, the mid-price will shift downward.[7]

At one extreme, a trader who must liquidate a large position can do so at a deep discount relative to the mar-

Exhibit 5

Probability of Default as a Function of Capital Ratio (m)

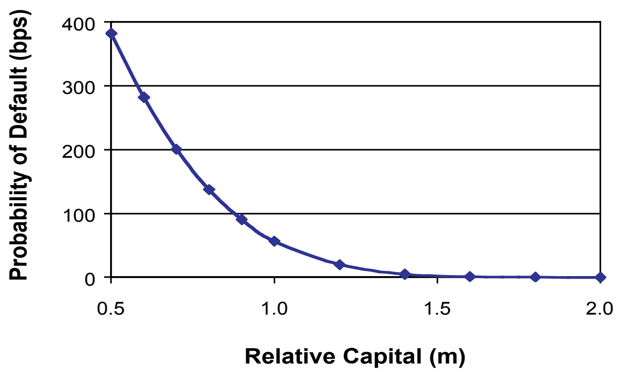
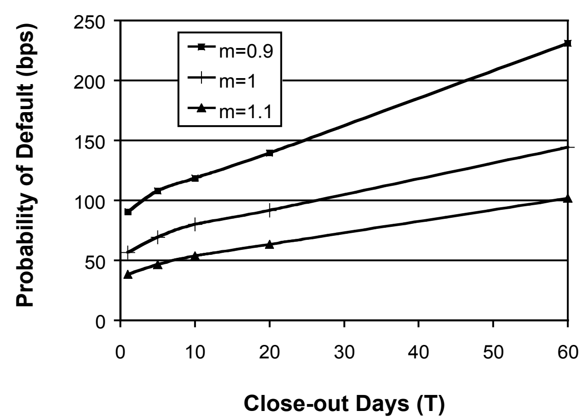


Exhibit 7

Illiquidity Increases the Probability of Default

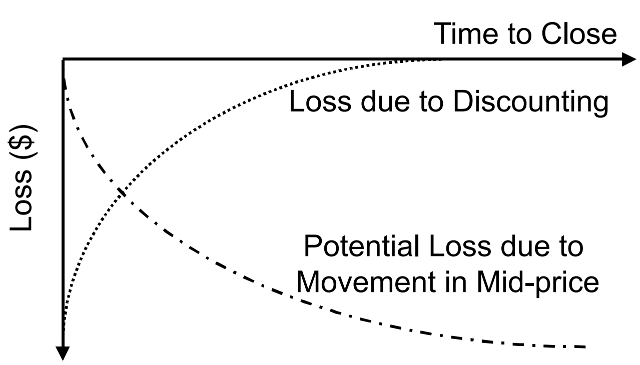


ket, and realize an immediate loss. At the other extreme, a trader can liquidate the position over an extended period of time, in smaller lots that are less likely to influence the market mid-price. During the extended liquidation period, however, the remaining position is exposed to additional potential losses. The trade-off between an immediate loss and losses over time is illustrated in Exhibit 6. In this article, we assume that the trader risks further losses by liquidating the position over time, rather than selling immediately at a deep discount.

For this simulation, liquidity was parameterized by the time (T) required to liquidate the entire position.[8] For example, if the position took ten days to close (T = 10), and the limits were reduced overnight by 20%, it would take two days for the trader to reduce the position to comply with the new limits.

Exhibit 6

Illustration of the Trade-Off Between Taking an Instant Discount or Being Exposed to Further Losses



The details of including liquidity in the program are in the appendix, and the results are illustrated in Exhibit 7. All lines show an increasing cumulative probability of default as the number of days to close the position increases. In other words, if exposure is reduced more slowly, the cumulative or conditional likelihood of experiencing a loss that reduces the level of capital below zero increases with time. In Exhibit 7, the top line illustrates the higher probabilities of default if capital is reduced by 10% (m = 0.9), the lowest line shows the lower probabilities of default if capital is increased by 10% (m = 1.1).

In this simulation, for an institution setting its limits according to current BIS capital requirements (m = 1), with a one-day close-out period, the probability of default is 57 bp. By contrast, over a ten-day close-out period, the probability of default increases to 80 bp as those positions realize interim losses. If a loss occurs in a liquid market, the trader can reduce the position immediately and only mitigate exposure to additional losses the next day. If the position takes ten days to liquidate, there is an increased probability of suffering additional large dollar losses, which may materially erode capital.

The probability of default based on a ten-day close-out period and m = 1 is approximately equivalent to the probability of default based on a forty-day close-out period and m = 1.1. From this observation, we conclude that a position that is liquidated over a forty-day period requires an additional solvency reserve of approximately 10% more capital than a position which is liquidated over a ten-day period. Later, we will use this observation to calculate the required capital ratio: α.

SPEED OF MANAGEMENT INTERVENTION

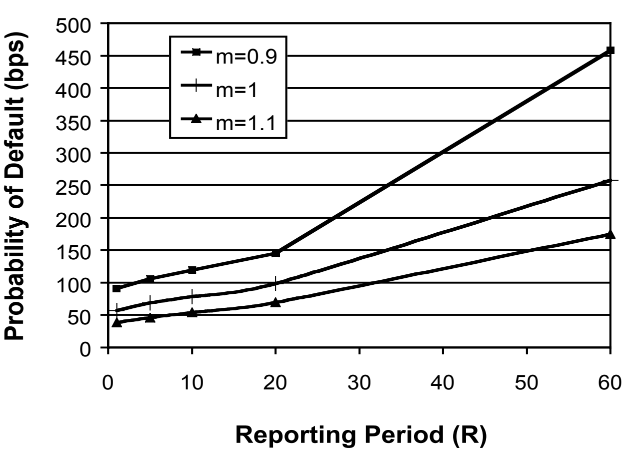
As a final level of complexity, we examine the effect of response lags to market changes. Slow responses to market changes increase risk. Ideally, in a well-run institution, risk and required capital are measured daily, and the risk management function will adjust trading limits each day. IT and internal political constraints, however, may make such responsiveness difficult to achieve in practice.

In this simulation, we add a response lag such that the limits are reviewed every R days, where R ranges from 1 to 250. With R = 1, the limits are revised daily. With R = 250, the limits are fixed at the beginning of the year and not revised until the end of the year. Exhibit 8 illustrates that the probability of default increases as the response time, R, increases. (This graph shows the results with the mild illiquidity of T = 10).

Note that if the limits are reviewed only annually (i.e., R = 250), we obtain the intuitive result that the probability of default is the same as for a completely illiquid position. In this case, a fully liquid position without management intervention carries the same risk as a fully illiquid position.

Exhibit 9 shows the probability of default for various combinations of close-out days and reporting periods. If the position is liquid and tightly managed (T = 1, R = 1) then the probability of capital falling below zero at the end of the year is 57 bp. If the position is illiquid (T = infinity), or if the limits are not changed during the year (R = 250), then the probability of default drastically increasing to 11.39%.

EXHIBIT 8
Slow Response Increases the Probability of Default



Let us consider the example where, on average, banks share the following common characteristics:

- A response period (R) of five days.
- Positions with an average close-out period (T) of ten days.
- Capital relative to VaR equal to the current BIS requirement (m = 1) as shown in Exhibit 9.

If this were the case, the annual average probability of default for a bank would be 84 bp.

APPLICATION OF THE RESULTS TO CAPITAL MANAGEMENT

So far, the results have shown the probability of default to be a function of capital, liquidity and response time. To illustrate an application to capital management, let us now reverse the problem by fixing the probability of default, and then determining how the capital ratio (m) must change to maintain that fixed level of creditworthiness given different combinations of liquidity (T) and response time (R).

Let us fix the desired probability of default to be 84 bp, well within the implied level of 1% imposed by the BIS (100% - 99%). For each combination of closeout time (T) and reporting period (R), we can vary m to find the amount of capital required such that the probability of default is 84 bp. The label α is used for the value of m that gives the required probability of default. The results are the values for a shown in Exhibit 1. The minimum value of a is 0.92 for a liquid position with daily limit resets. The maximum value is 1.79 for a completely illiquid portfolio (e.g., a portfolio in which trading limits are reset annually.)

EXHIBIT 9
Probability of Default (bp) for Combinations of R and T (m - 1)

Reporting Period (R)	Closeout Days (T): 1	5	10	21	62	∞
1	57	69	80	92	144	**1139**
5	68	78	84	99	158	**1139**
10	78	85	94	109	176	**1139**
21	99	105	113	129	237	**1139**
62	258	288	319	375	535	**1139**
250	**1139**	**1139**	**1139**	**1139**	**1139**	**1139**

CONCLUSION

In conclusion we wish to convey three points:

- Banks should be required to modify their capital according to the liquidity of the position and the effectiveness of limit management policies.
- The amount by which capital is modified, can be set according to a published table (as in Exhibit 1), or according to internal models of the institution's trading process, and
- Regardless of whether or not such an approach is adopted by regulators, internal managers can use this approach for capital-setting and differentiating capital charges according to liquidity.

The fundamental principle is that closely-managed, liquid positions should require less capital than loosely-managed, illiquid positions.

APPENDIX

Description of the Basic Approach

Exhibit A shows the layout of the basic approach. It does not include the effects of liquidity or delays in resetting limits and the allowed position.

Incorporation of Illiquidity

In the case illustrating a portfolio of liquid instruments (Exhibit A), we assume that changes in the position size each day completely reflect changes in the limits (which in turn are related to the available capital). The effect of illiquidity is incorporated by imposing a restriction on the maximum amount by which the position can be changed each day. In this model, the maximum amount is set to be the position size divided by the close-out period. In a more complex model, the maximum amount could be set as a fraction of the daily market traded volume.

The reduction in position each day is therefore the minimum of 1) the desired change in position or 2) the maximum amount that can be traded by the bank in one day:

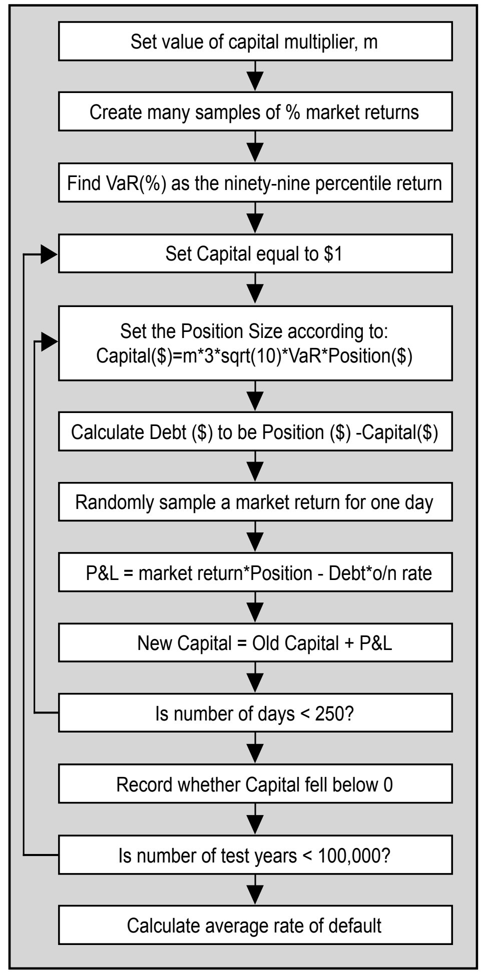
Reduction = min(desired amount, max amount)

Incorporation of Delays

Delays are incorporated by only resetting the limits (and therefore the desired position) every R days.

EXHIBIT A

Detailed Flowchart for the Basic Model



The Stochastic Simulation of Market Returns

The daily returns on the S&P 500 index were estimated over a thirty-year period (1970-present). Based on the following parameters for S&P returns the random daily P&L was simulated employing a jump-diffusion process approach:[9] a) mean of 0.00037, b) standard deviation of 0.009651, and c) kurtosis of 30.417.[10]

For each time step of the simulation, with a) a 99.9% probability an observation would be drawn from a distribution with a standard deviation of 0.09 and b) a 0.1% probability that an observation would be drawn from a normal distribution with a standard deviation of 0.1.

After each draw was made, the value 0.00037 was added to the observed result.

Cost of Debt

The approach assumes that the position is funded by a mixture of equity capital and debt. Daily interest payments are made on the outstanding debt. This model fixes the cost of debt at 6% per year. In a more complex model, the debt rate could be correlated with the returns on the market. (Historically there has been a correlation of around 0.2 between changes in the S&P 500 and changes in interest rates.) An even more sophisticated approximation may assume interest rates to be mean reverting with random shocks that are correlated with market returns.

Additions and Refinements

The example above is deliberately stylized to ensure that the key points are not obscured by technical details. Further refinements may impact the estimates of the probability of default, as well as the values exhibited in Exhibit 1. The relative order of values in Exhibit 1, and hence its implications, should not change.

Volatility Clustering. The basic model described above assumes that the parameters of the market are stationary and therefore that the VaR is constant. A more precise model specification would allow for non-constant volatility similar to the GARCH family of models.

Correlation of Liquidity with Losses. The approach described above assumes that the closeout period for a particular position or instrument remains constant, i.e., independent of market conditions. Modeling the correlation between market movements and liquidity, e.g., a severe negative shock in the market may also reduce liquidity, and thereby increase the potential loss suffered before the position is closed out. This effect is likely to increase the estimated probability of default, but is not expected to significantly change the values for α in Exhibit 1.

Variability in the Bid-Ask Spread. In this framework, it is possible to account for both risks associated with movements in the mid price and the risks associated with changes in the bid-ask spread. One may assume that changes in the bid-ask spread are perfectly correlated with market returns by simply increasing the volatility of returns to include the volatility of the spread (as discussed in Bangia, Diebold, Schuermann, and Stroughair [1999]). Alternatively, one could simulate the bid-ask spread as a separate risk factor, correlated with changes in the mid-price. Depending upon the magnitude of the correlation coefficient, the risks exhibited by the latter approach might be significantly lower.

Fixed Income Markets. This example samples the portfolio P&L from an underlying distribution with characteristics similar to those observed in the equity markets. The P&L process for a fixed-income or bank loan portfolio should instead be modeled according to a mean-reverting stochastic interest rate model and employ bond pricing analytics to compute portfolio values.

Positions Below the Limit. The proposed model assumes that traders will completely consume the maximum allowed limits, on the theoretical basis that traders are typically motivated by profit incentives to take as much risk as the limits allow. In practice, traders often exhibit risk averse behavior and thereby choose to have less than maximum exposure. The observed behavior could be simulated in the model by treating the trader's position as a random, varying percentage of the allowed position. Further, sensitivity analyses can be performed based on the parameters of this distribution (such as uniform, normal, mean, and standard deviation)

Multiple Close-Out Periods. The approach assumes that all positions in the institution have the same close-out period. In practice, this is not likely to be the case. One of two approaches can be employed to account for cross-sectional differences in liquidity between positions in different markets: a) simply compute a weighted average of the closeout periods (weighted by the relative size of the positions) or b) explicitly model each position with randomly occurring states of liquidity.

Results

Exhibit B shows Exhibit 1 from the main text, but with all elements divided by the value 0.92. Exhibit C is derived in the same way as Exhibit 1 except that the probability of default for Exhibit 1 is 84 bp and for Exhibit C it is fixed at 57 bp, which

EXHIBIT B
Relative Capital for T = 10 and R = 5

Reporting Period (R) \ Closeout Days (T)	1	5	10	21	62	∞
1	1	1.04	1.07	1.10	**1.23**	**1.89**
5	1.04	1.08	1.08	1.13	**1.23**	**1.89**
10	1.07	1.08	1.11	1.15	**1.24**	**1.89**
21	1.13	1.14	1.16	1.19	**1.27**	**1.89**
62	**1.28**	**1.28**	**1.28**	**1.28**	**1.44**	**1.89**
250	**1.89**	**1.89**	**1.89**	**1.89**	**1.89**	**1.89**

EXHIBIT C
Relative Capital for T = 1 and R = 1

Reporting Period (R) \ Closeout Days (T)	1	5	10	21	62	∞
1	1	1.04	1.08	1.11	**1.21**	**1.96**
5	1.04	1.07	1.09	1.13	**1.22**	**1.96**
10	1.07	1.09	1.12	1.15	**1.27**	**1.96**
21	1.13	1.15	1.16	1.19	**1.32**	**1.96**
62	**1.32**	**1.32**	**1.32**	**1.32**	**1.50**	**1.96**
250	**1.96**	**1.96**	**1.96**	**1.96**	**1.96**	**1.96**

is the probability of default with m = 1, R = 1, and T = 1. Notice that there is very small difference between Exhibits B and C, i.e., the capital ratio is not sensitive to the combination of closeout and delay chosen as the "average bank" base-case.

ENDNOTES

[1]The factor may be greater than 3, if the VaR calculator performs poorly on backtesting.

[2]For example, with a reporting period (R) of five days, as the close-out days change from 1 to 21 the relative capital increases from 0.95 to 1.04. This 9% increase is much less than the square root of 21. The "Root T" approach describes the loss the bank may suffer if it ordered all positions to be closed out. This is not how trading operations work. In practice, during the T days taken to close an illiquid position, a trader in an otherwise identical but liquid market could have taken T different bets (within the limit structure) and lost on each one, thereby losing almost as much as the illiquid position. The consequence is that the "Root T" approach overstates the relative risk of illiquid positions. For further critical assessment of the "Root T" approach, see Diebold, Hickman, Inoue, and Schuermann [1998].

[3]The probability of default may be significantly different for other types of limit structure, e.g., traditional max contract size combined with stop-loss. When using the approach of this paper, a bank should incorporate the policies that it actually uses.

[4]It is not critical to recreate the skew of market returns because the effect on P&L will depend on whether the position is long or short the market. The skewness can be recreated using bootstrap resampling, but this introduces limitations to the number of simulations that can be meaningfully run.

[5]We assume that each day the value (V) of the position is funded by a combination of capital (C) and debt (D): $V = C + D$. The net profit (P) each day is the market return (r) minus the cost of debt: $P = Vr - Di$, where i is the overnight cost of debt. In this example the cost is fixed at 6%/250. A more complex model could have the cost of debt as a mean reverting function correlated with the market.

[6]See Bangia, Diebold, Schuermann, and Stroughair [1999].

[7]See Chriss and Almgren [1998].

[8]It is difficult to estimate the number of days required to close a position. One objective approach is to use the following formula to classify the liquidity of different positions:

Closeout Days =
Position Size/F × Daily Market Traded Volume

where F is a factor that is decided and fixed a priori for the whole institution. F = 0.1 would imply that 10% of the daily volume can be sold each day without significantly shifting the market. The "Daily Market Traded Volume" can be the average volume or the volume in a crisis period, e.g., the average minus two standard deviations.

[9]See Akgiray and Booth [1987]

[10]The results presented here are based on returns computed using absolute percentage as opposed to relative (log) percentage changes. The estimated moments, which drive the simulation, using the log metric differ only slightly.

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